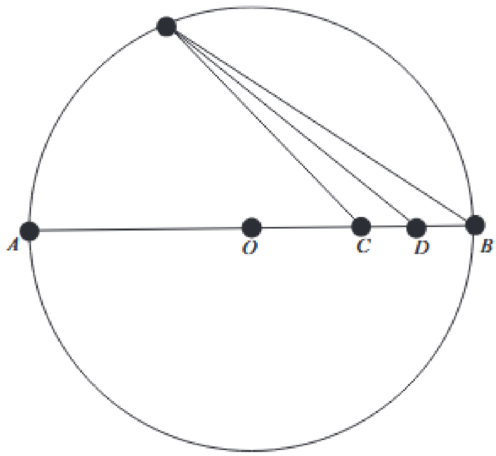
Problems from the College Mathematics Journal, due September 15, 2026

1321. *Proposed by Yagub Aliyev, ADA University, Baku, Azerbaijan.*

A circle with diameter AB, points C and D on diameter AB, and point E on the circle are given such that point D is closer to point B than C is and $E \neq A, B$. Prove that $\frac{\angle DEB}{\angle CEB} > \frac{BD}{AD} \cdot \frac{AC}{BC}$.



1322. *Proposed by Mohammad Azarian, University of Evansville, Indiana.*

Let F_n and L_n ($n \geq 1$) be arbitrary Fibonacci and Lucas numbers, respectively. Also, let

$$A = \left(\sum_{i=0}^{\infty} \frac{1}{i!}\left(1 + \sum_{j=1}^{n} F_j\right)^i\right)\left(\sum_{i=0}^{\infty} \frac{1}{i!}\left(\frac{F_{2n}}{L_n}\right)^i\right)^3 \left(\sum_{i=0}^{\infty} \frac{1}{i!}\left(\frac{\sum_{j=1}^{n} F_j^2}{F_n}\right)^i\right)^2,$$

$$B = \left(\sum_{i=0}^{\infty} \frac{1}{(2i)!} F_{n+2}^{2i}\right)\left(\sum_{i=0}^{\infty} \frac{1}{(2i+1)!} F_{n+2}^{2i+1}\right)\left(\sum_{i=0}^{\infty} \frac{(-1)^i}{i!} F_n^i\right).$$

Determine the coefficient of F_{n+2}^k in the expansion of AB, for all $k \geq 0$.

1323. *Proposed by Ovidiu Furdui, Technical University of Cluj-Napoca, Romania.*

Let a and b be positive real numbers. Calculate (with proof)

$$\lim_{n\to\infty} \int_0^1 \sqrt[n]{a(-\ln x)^n + b}\, \mathrm{d}x.$$

1324. *Proposed by Sela Fried, Israel Academic College, Israel.*

Let n be a positive integer. In how many ways can we write the numbers $1, 2, \ldots, 2n$ as the elements of a $2 \times n$ matrix such that no two consecutive numbers are in the same column?

1325. *Proposed by Tuan Anh Nguyen, Nguyen Quang Dieu High School for the Gifted, Vietnam.*

We denote the set of natural numbers $\{0, 1, 2, \ldots\}$ by $\mathbb{N}$. A set $S \subseteq \mathbb{N}$ is called **good** if it satisfies simultaneously the following two conditions:

1. $\forall a, b \in S, a \neq b \Longrightarrow \gcd(a, b) > 1$;
2. $\forall a, b \in S, a \neq b \Longrightarrow a \nmid b$ and $b \nmid a$.

For each $n \in \mathbb{N}$, we define

1. $a_n = 0$ when $n = 0, 1$. For $n \geq 2$, let $\mathcal{S}_n = \{S \subseteq \{1, 2, \ldots, n\} \mid S \text{ is a good set}\}$ and define $a_n = \max_{S \in \mathcal{S}_n} |S|$.
2. $b_n = \operatorname{sgn}((n-5)(\operatorname{mod} 3)) - \operatorname{sgn}((n-5)(\operatorname{mod} 2))$, where $\operatorname{sgn}\colon \mathbb{Z} \to \{-1, 0, 1\}$ is given by

$$\operatorname{sgn}(x) = \begin{cases} -1, & \text{if } x < 0, \\ 0, & \text{if } x = 0, \\ 1, & \text{if } x > 0. \end{cases}$$

Prove that for all $n \in \mathbb{N}$, we have

$$\sum_{k=0}^{n} a_k b_{n-k} = t_n,$$

where t_n is the nth Alcuin number, defined as the coefficients in the power series expansion of the following rational function:

$$\sum_{n=0}^{\infty} t_n X^n = \frac{X^3}{\left(1 - X^2\right)\left(1 - X^3\right)\left(1 - X^4\right)}.$$

Problems from the American Mathematical Monthly, due September 30

12601. *Proposed by Tuan Anh Nguyen, Nguyen Quang Dieu High School for the Gifted, Dong Thap, Vietnam.* How many subsets of $\{1, \ldots, n\}$ contain exactly one pair $\{a, b\}$ of distinct elements satisfying $n \le a + b \le n + 1$?

12602. *Proposed by Yun Zhang, Xi'an, China.* The *Brocard angle* of a triangle XYZ is the unique angle ω such that there exists a point P inside the triangle with $\angle PXY = \angle PYZ = \angle PZX = \omega$. Given $\triangle ABC$, choose D, E, F on BC, CA, AB, respectively, such that $BD/DC = CE/EA = AF/FB$. Prove that the Brocard angles of $\triangle ABC$ and $\triangle DEF$ are equal.

12603. *Proposed by Daniel Reeves, Beeminder, Portland, OR.* For a positive integer k, let $2{\uparrow\uparrow}k$ denote the exponential tower consisting of k copies of the number 2. For example, $2{\uparrow\uparrow}4 = 65536$. For a positive integer n and a large positive integer k, one might try to compute the n rightmost decimal digits of $2{\uparrow\uparrow}k$ by repeated exponentiation but truncating the numbers after each step, keeping only the n rightmost decimal digits. For which n does this approach give the correct answer for all k? In other words, let $s_1 = 2$, and for $k \ge 2$, let s_k be the least nonnegative residue of $2^{s_{k-1}}$ modulo 10^n. For which values of n is s_k congruent to $2{\uparrow\uparrow}k$ modulo 10^n for all k?

12604. *Proposed by Erik Vigren, Swedish Institute of Space Physics, Uppsala, Sweden.* For positive integers k and m, we say that k is *m-avoiding* if there is no integer n with $m^2 \le n < k$ such that either $k^2 < mn^2 < (k+1)^2$ or $k^2 < mn(n+1) < (k+1)^2$.
(a) Find all 2-avoiding integers, all 3-avoiding integers, and all 4-avoiding integers.
(b) Are there infinitely many 5-avoiding integers?

12605. *Proposed by Aryan Desai (student), Ranchhodlal Chhotalal Technical Institute, Ahmedabad, India.* Evaluate

$$\int_0^{\pi} \arccos\left(\frac{\sqrt{3}\cos x + \sin x - 2}{\sqrt{7 - 4\sqrt{3}\cos x}}\right) dx.$$

12606. *Proposed by Mohammadreza Ghelichkhani, Shahed University, Tehran, Iran.* Call a polynomial P with integer coefficients *special* if $P(a)$ is a nonsquare for every $a \in \mathbb{Z}$ but for every positive integer n there exists $a \in \mathbb{Z}$ such that $P(a)$ is a square modulo n.
(a) Is there a linear polynomial that is special?
(b) Is there a quadratic polynomial that is special?
(c) Is there a cubic polynomial that is special?

12607. *Proposed by Vasile Cîrtoaje, Petroleum-Gas University of Ploieşti, Ploieşti, Romania.* What is the smallest positive real number p such that

$$(ab)^p + (ac)^p + (ad)^p + (bc)^p + (bd)^p + (cd)^p \ge 6$$

for all real numbers a, b, c, d with $a \ge b \ge c \ge d \ge 0$ and $ab + bc + cd + da = 4$?

Problems from the American Mathematical Monthly, due October 31

12608. *Proposed by Nikolai Osipov, Siberian Federal University, Krasnoyarsk, Russia.* Find all triples (x, y, z) of positive integers such that $x^3 - y^3 = (xyz - 1)^2$.

12609. *Proposed by Hüseyin Yiğit Emekçi, İinzmir, Turkey.* Let $a_1, \ldots, a_n$ be nonnegative real numbers, and for a positive integer m define $r_m = \sum_{i=1}^{n} a_i^m$. Prove

$$r_5 + 2r_4 + r_3 \geq r_1 \left(2r_3 + 2r_2 - r_1^2\right),$$

and determine when equality holds.

12610. *Proposed by Christopher D. Long, Lexington, KY.* Prove

$$\sum_{k=1}^{2n} (-1)^k \frac{\sin\big(k(kz+2)\pi/(nz+1)\big)}{\sin\big(kz\pi/(nz+1)\big)} = n$$

for every positive integer n and every complex number $z \in \mathbb{C}$ such that the sum is defined.

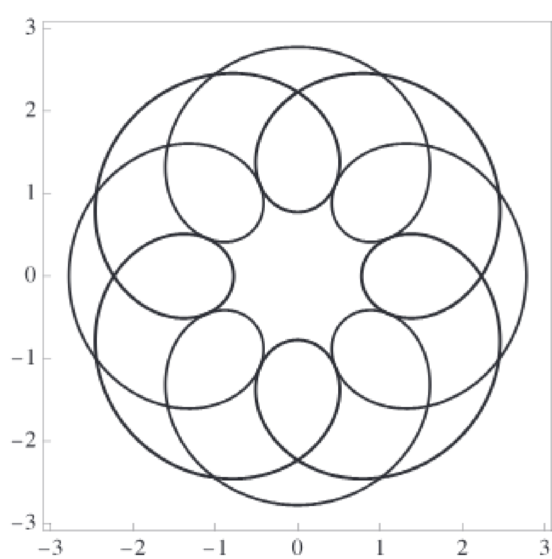
12611. *Proposed by Vladimir Lucic, Imperial College, London, UK.* For a real number c, say that a square matrix A of real numbers is *c-rigid* if $A_{i,1} = c$ for all i and $A_{i,j} = 0$ for all i and j with $j \geq i+2$. Show that for every real symmetric positive-definite matrix M, there is a unique positive value of c such that M can be written in the form PAA^TP^T for a permutation matrix P and a c-rigid matrix A.

12612. *Proposed by Marián Štofka, Bratislava, Slovakia.* Let

$$I_{a,b} = \int_0^1 \frac{1}{x}\big(\ln(1-x)\big)^a \big(\ln(1+x)\big)^b \, dx,$$

and let ζ be the Riemann ζ-function. Prove $48I_{5,1} + 160I_{3,3} + 48I_{1,5} = -4275\zeta(7)$.

12613. *Proposed by Alexander Bousman, Greg Dresden, Anshika Patel, Ben Rubin, and Sam Wizda, Washington and Lee University, Lexington, VA.* For a positive integer n with $n \geq 2$, let a_n be the smallest positive value of a that yields tangency of the loops of the epitrochoids $(a\cos t + \cos(nt), a\sin t + \sin(nt))$ and $(a\cos t - \cos(nt), a\sin t - \sin(nt))$. The figure shows the case of $n = 5$, where a_5 is approximately 1.77.



(a) Prove that a_n is an algebraic number for all n.

(b) Prove that $\lim_{n\to\infty} a_n/n$ equals $\cos c$, where c is the smallest positive solution to $\tan c = c + \pi/2$.

12614. *Proposed by Navid Safaei, Bulgarian Academy of Sciences, Sofia, Bulgaria.* For which rational functions $R(x)$ with real coefficients does there exist a polynomial $P(x)$ with real coefficients such that $R(x) - R(x+1) = R(P(x))$?

Same Circumcenters (P502). Friday Michael (Toronto, Canada) poses this problem.
Consider the triangle ABC with $|CA| = |AB|$ and circumcenter O. Choose points B_1 and B_2 on CA along with points C_1 and C_2 on AB such that $|B_1C_1| + |B_2C_2| = |BC|$ and B_1C_1, B_2C_2, and BC are parallel. A triangle is then chosen from the set of nondegenerate triangles that can be formed from the points $A, C_1, C_2,\ B,\ C, B_2,$ and B_1.

What is the probability that the triangle has circumcenter O?

Two Solvable Collatz-like Maps (P503). Jordan Hess (University of Texas at Dallas) brings readers this problem. Let P be the set of positive integers. Consider the two maps from P to P defined by

$$T_1(n) = \begin{cases} \dfrac{3n+1}{4} & \text{if } n = 1 \pmod 4 \\ \dfrac{3n-1}{4} & \text{if } n = 3 \pmod 4 \\ \dfrac{n}{2} & \text{if } n = 0 \pmod 2 \end{cases}$$

and

$$T_2(n) = \begin{cases} \dfrac{3n-1}{2} & \text{if } n = 1 \pmod 4 \\ \dfrac{3n+1}{2} & \text{if } n = 3 \pmod 4 \\ \dfrac{n}{2} & \text{if } n = 0 \pmod 2 \end{cases}.$$

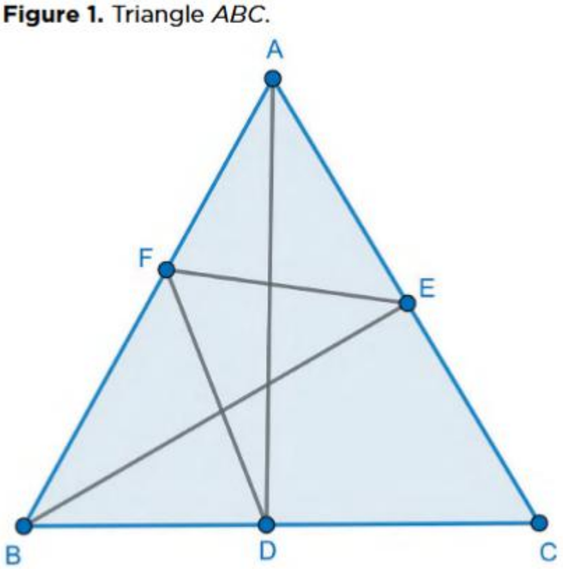
For each $a \in P$, generate a sequence from a by iteratively applying one of the T_i. Prove that sequences eventually reach 1 when using T_1 for every $a \in P$, but sequences using T_2 eventually reach 1 only when a is a power of 2 and diverge otherwise.

From Zero to Hero (P504). Robert Brignall (Open University, UK) and Vincent Vatter (University of Florida) suggested this problem connected to their article "Digit Party: Three Years of Lying About High Scores." As described in the article, Digit Party consists of placing 25 digits from 1 to 9 in a 5×5 grid where the digits are random and arrive one at a time. When two adjacent cells (horizontally, vertically, or diagonally) hold the same digit, the player scores points equal to that digit's value. For example, a pair of neighboring 7s is worth seven points.

Call a Digit Party board *zero-able* if its 25 digits can be arranged on the grid so that no two adjacent cells hold the same digit, scoring zero points. Among all zero-able boards, which one has the highest maximum score when the digits are played optimally, and what is that score?

Triangular Areas Inequality (P505). Yagub Aliyev (ADA University, Azerbaijan) proposed this problem. In the triangle ABC shown in figure 1, the points D, E, and F lie on BC, AC, and AB, respectively. Prove that if $n^2 > n$, then
$$[AEF]^n + [BDF]^n \geq \left([ABE]^{n/(1-n)} + [ABD]^{n/(1-n)}\right)^{1-n}$$
where $[XYZ]$ denotes the area of triangle XYZ. Furthermore, prove that if $n^2 < n$, then the reverse inequality holds. When does one obtain equality?

Figure 1. Triangle *ABC*.

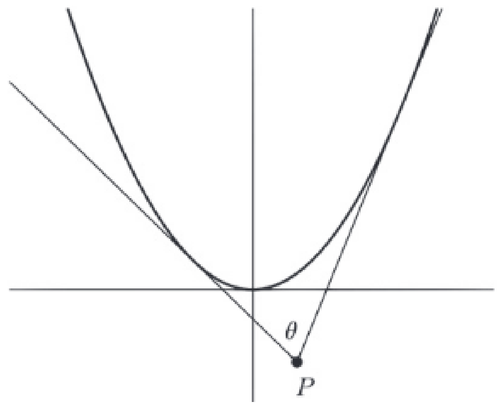


Problems from Mathematics Magazine, due November 1, 2026

2246. *Proposed by Matt Wright, Missouri State University, Springfield, MO.*

Fix $\theta \in (0, \pi)$. Find the locus of points P in the plane with the property that there exists an angle with vertex P and measure θ such that both sides of the angle are tangent to the parabola $y = x^2$.

Note: This is a variation on a problem from the 2026 Missouri Collegiate Mathematics Competition.



2247. *Proposed by Hideyuki Ohtsuka, Saitama, Japan.*

Let t_n be the Thue-Morse sequence, defined by $t_{2n} = t_n$ and $t_{2n+1} = 1 - t_n$ for any integer $n \geq 0$, with $t_0 = 0$. Evaluate

$$\sum_{n=1}^{\infty} \left(\frac{1}{4n} + \frac{1}{4n+1} - \frac{1}{4n+2} - \frac{1}{4n+3} \right) t_n.$$

2248. *Proposed by Antonio Garcia, Strasbourg, France.*

Let

$$I_n(a,b) = \int_0^{\infty} \frac{\ln(x+1)}{\sqrt{x}(1+ax)^n(1+bx)^n}\, dx.$$

Show that asymptotically

$$I_n(a,b) \sim \frac{\sqrt{\pi}}{2(a+b)^{\frac{3}{2}} n^{\frac{3}{2}}}$$

as $n \to \infty$.

2249. *Proposed by Mátyás Barczy, University of Szeged, Szeged, Hungary.*

Choose a real number a uniformly from the interval $[-\lambda, \lambda]$. What is the probability that the curves

$$y = -ax^2 + (a-1)x \quad \text{and} \quad y = x^3 + 1$$

intersect in three distinct points?

2250. *Proposed by Harry Hutchins, Dekalb, IL.*

Call a subgroup of $\mathbb{R} - 0$ under multiplication *excellent* if it is also closed under addition. It is easy to see that $\mathbb{Q}^+$ and $\mathbb{R}^+$ are excellent.

(a) Find an excellent subgroup strictly between $\mathbb{Q}^+$ and $\mathbb{R}^+$

(b) Find an excellent subgroup that does not consist solely of positive numbers.

Problems from the College Mathematics Journal, due November 15, 2026

1326. *Proposed by Michael Weselcouch, Roanoke College, Salem, VA.*

Let S_n be the set of all permutations of $[n] = \{1, 2, \ldots, n\}$. For $\pi \in S_n$, an entry k is a *dominant value* of π if $k > n/2$ and all lower values appear to the right of k in the permutation. (Thus permutations are represented consistently with one-line notation.) Let $\mathrm{dv}(\pi)$ be the number of dominant values of π. For example, if $\pi = 43512$, then the values 4 and 3 are dominant values, but not 5 (lower values to its left) or 2 (too small).

Evaluate, with justification,

$$\lim_{n\to\infty} \frac{1}{n!} \sum_{\pi \in S_n} \mathrm{dv}(\pi).$$

1327. *Proposed by Stanescu Florin, Serban Cioculescu, Gaesti, Romania.*

If $f : (0, \infty) \to \mathbb{R}$ is a convex function, show that

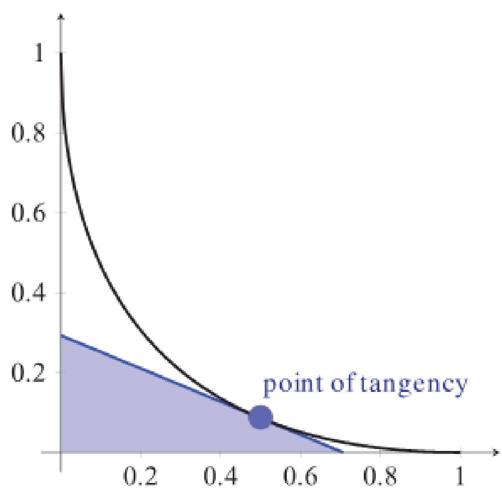
$$\lim_{n\to\infty} \int_{2(n+1)\pi}^{2n\pi} f(x)\sin(x)\,dx = 2\pi \cdot \sup_{n\in\mathbb{N}} \frac{f(2n) - f(n)}{n}.$$

1328. *Proposed by Gregory Dresden, Washington & Lee University, Lexington, VA.*

For s, t positive integers, find the maximum possible area for a triangle bounded by the coordinate axes and a line tangent to

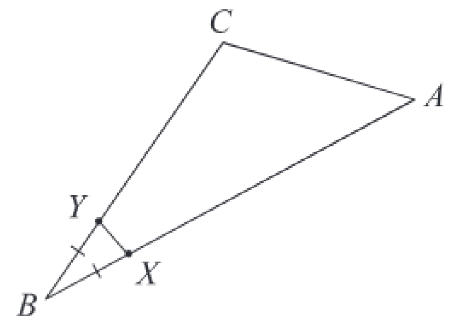
$$x^{1/s} + y^{1/t} = 1,$$

as shown here.



1329. *Proposed by Jason Zimba, Amplify Education, NY and Claire Zimba, Bronx High School of Science, NY.*

The ratio perimeter^2/area for a plane figure is called the *isoperimetric ratio*. For a triangle ABC, define the process of *trimming the triangle at vertex B* as the removal of an isosceles triangle XBY, where X is a point on side AB and Y is a point on side BC, resulting in a quadrilateral $AXYC$. Segment XY is called the *cut*. Show that the isoperimetric ratio of the quadrilateral is minimized when we trim the triangle at a vertex of smallest (or minimal) angle, with a cut that runs tangent to the incircle of the triangle.



1330. *Proposed by Tran Quang Hung, High School for Gifted Students, Vietnam National University, Hanoi, Vietnam.*

Given a triangle ABC, construct a rectangle $BCMN$ external to it. Let K be a point such that $KM \perp AC$ and $KN \perp AB$. Denote by H the orthocenter of triangle ABC. Let the perpendicular bisector of segment BN intersect line BH at S, and let J be the midpoint of AH. Construct rectangle $SJPT$ such that P lies on AC. Prove that the lines KT and BC are parallel. (See Figure 1.)

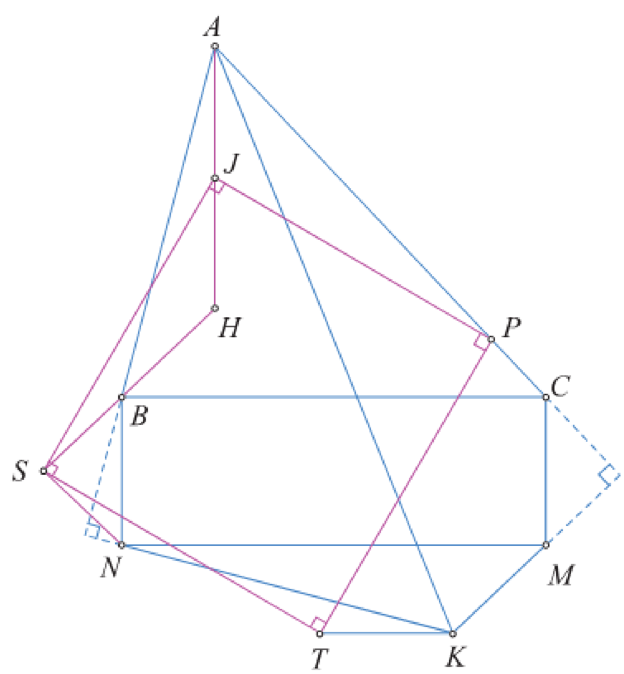


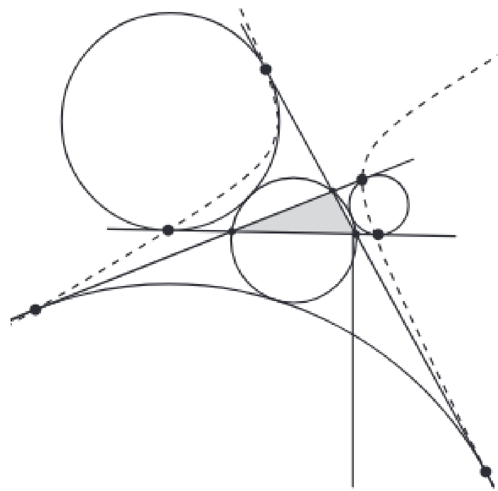
Figure 1. Figure for Problem 1330.

Problems from the American Mathematical Monthly, due December 31

12615. *Proposed by Eric Bach, University of Wisconsin, Madison, WI, and Jeffrey Shallit, University of Waterloo, Waterloo, Canada.* A polynomial $\sum_{i=0}^{n} c_i x^i$ with coefficients c_i in a field F and with $c_n \neq 0$ is called *palindromic* if $c_i = c_{n-i}$ for all $i \in \{0, \ldots, n\}$. Show that all polynomials over F can be written as the sum of at most three palindromic polynomials.

12616. *Proposed by Tho Nguyen Xuan, Hanoi University of Science and Technology, Hanoi, Vietnam.* Let $m = n^2 + n + 41$ for some integer n. Show that for any positive divisor d of m, either d or m/d has the form $u^2 + uv + 41v^2$ for some integer u and some odd integer v.

12617. *Proposed by Aryan Desai, Ahmedabad, India.* Given a triangle ABC, let Ω_A, Ω_B, and Ω_C be the circles externally tangent to the circumcircle of ABC and internally tangent to the angles at A, B, and C, respectively.



(a) Show that the six points of tangency between Ω_A, Ω_B, and Ω_C and the lines AB, BC, CA lie on a conic.

(b) Show that the conic is a hyperbola when $\triangle ABC$ is obtuse.

(c)* For which triangles is the conic an ellipse?

12618. *Proposed by Wanlong Han, Henan, China.* With $x_n = 6\left(3^{3^n} + 1\right)^2$, evaluate

$$\lim_{n\to\infty} \sqrt[3]{x_1 + \sqrt[3]{x_2 + \sqrt[3]{x_3 + \cdots + \sqrt[3]{x_n}}}}.$$

12619. *Proposed by Victor Miller, El Granada, CA, and Daniel J. Velleman, Amherst College, Amherst, MA.* A clock has hour, minute, and second hands, but the clock face is just a blank disk, with no numbers or markings on it. Furthermore, the clock has been rotated an unknown amount. The only data available to a viewer of the clock are the angles between the hands, which we assume can be measured with perfect accuracy.
(a) Show that if the hour, minute, and second hands are distinguishable, then it is always possible to determine the time.
(b) Show that if the hour, minute, and second hands are indistinguishable, then it is not always possible to determine the time.
(c) Suppose the three hands are indistinguishable. Is the set of times at which it is impossible to determine the time finite?

12620. *Proposed by Andrei Jorza, University of Notre Dame, Notre Dame, IN.* Let $\Phi_n(x)$ denote the nth cyclotomic polynomial.
(a) Show that if $x^m + \Phi_n(x)$ is irreducible for all even m, then $n = p^k$ or $2p^k$, where p is a Mersenne prime and k is a nonnegative integer.
(b) Show that $x^m + \Phi_{3^k}(x)$ and $x^m + \Phi_{2\cdot 3^k}(x)$ (which equal $x^m + x^{2\cdot 3^{k-1}} \pm x^{3^{k-1}} + 1$) are irreducible for all even m and positive integers k.

12621. *Proposed by Mohammed Aassila, Strasbourg, France.* Let $\{\{a_n, b_n\} : n = 1, 2, \dots\}$ be a partition of $\mathbb{Z}$ with $a_n < b_n$ and $a_n + b_n = n$ for all n. Prove that there are infinitely many n such that $b_n \geq (13/7)n$.